Worcester County Mathematics League

Varsity Meet 3 - January 22, 2020

COACHES' COPY
ROUNDS, ANSWERS, AND SOLUTIONS

Worcester County Mathematics League
Varsity Meet 3 - January 22, 2020
Answer Key

Round 1 - Similarity and Pythagorean Theorem

1. $\frac{20}{3}$ or $6\frac{2}{3}$ or $6.6\overline{6}$
2. $\sqrt{65}$
3. $1:5$ or $\frac{1}{5}$ or 0.2

Round 2 - Algebra I

1. 9
2. $(4, 6)$ and $(-4, 10)$
3. 5 and 1

Round 3 - Functions

1. 64
2. $-\frac{7}{3}$ (note: negative) or $-2\frac{1}{3}$ or $-2.3\overline{3}$
3. 12

Round 4 - Combinatorics

1. -1
2. 45
3. 45

Round 5 - Analytic Geometry

1. $\frac{29}{5}$ or $5\frac{4}{5}$ or 5.8
2. $(-5, -9)$
3. 0 and -10

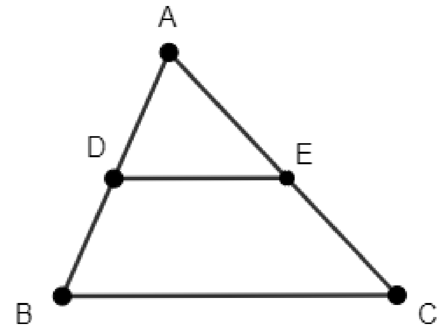
Team Round

1. $\frac{9}{25}$ or 0.36
2. 5
3. 15
4. 10
5. $\frac{3-\sqrt{5}}{2}$
6. $\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}$
7. $m + 7$
8. 2
9. 8

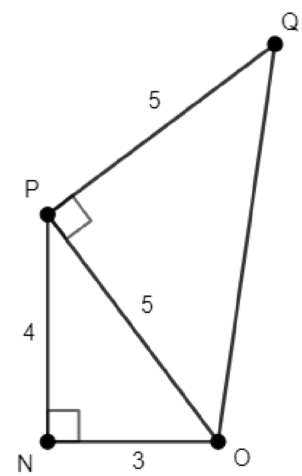
Worcester County Mathematics League
Varsity Meet 3 - January 22, 2020
Round 1 - Similarity and Pythagorean Theorem

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

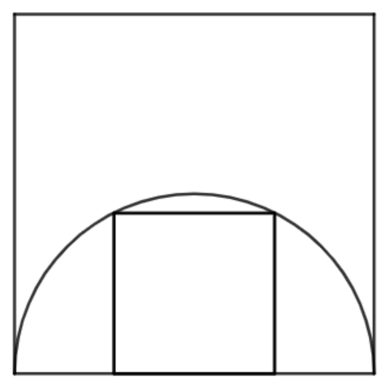
1. Given ΔABC with $\overline{DE} \parallel \overline{BC}$, $AB = 8$, $AD = 6$, and $DE = 5$, find BC.



2. Given the two right triangles in the picture shown to the right, how far apart are points N and Q?



3. A square is inscribed in a semicircle which is inscribed in a larger square. Find the ratio of the smaller square's area to the larger square's area.



ANSWERS

(1 pt) 1. ____________________

(2 pts) 2. ____________________

(3 pts) 3. ____________________

Southbridge, Shrewsbury, Worcester Academy

Worcester County Mathematics League
Varsity Meet 3 - January 22, 2020
Round 2 - Algebra I

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Solve for x: $\sqrt{4x-11} = 2\sqrt{x} - 1$

2. Find all ordered pairs that satisfy the system below.

$$\begin{cases} \dfrac{2}{x} + \dfrac{1}{y} = \dfrac{x}{y} \\ \\ 2y + x = 16 \end{cases}$$

3. Solve for x: $|x^2 + 3x - 10| = 6x$.

ANSWERS

(1 pt) 1. $x =$ ________________

(2 pts) 2. ________________

(3 pts) 3. $x =$ ________________

Westboro, Shrewsbury, Shrewsbury

Worcester County Mathematics League
Varsity Meet 3 - January 22, 2020
Round 3 - Functions

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. If $f(x) = 3x + 5$ and $g(y) = 4 - y$, determine the value of

$$(f(3) - g(-2))^2$$

2. Consider linear functions $f(x)$ and $g(x)$.

- $f(x)$ has a slope of $\frac{2}{3}$
- $g^{-1}(x) = 3f(x) - 1$
- $g(6) = 8$

Find $f(1)$.

3. Function f has the property that for any two positive real numbers x and y that

$$f(xy) = f(x) + f(y).$$

Function g takes only positive values and has the property that for any real numbers x and y that

$$g(x + y) = g(x) \cdot g(y).$$

If $f(g(1)) = 3$, compute $f(g(4))$.

ANSWERS

(1 pt) 1. ______________________

(2 pts) 2. ______________________

(3 pts) 3. ______________________

Bartlett, Shrewsbury, Algonquin

Worcester County Mathematics League
Varsity Meet 3 - January 22, 2020
Round 4 - Combinatorics

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Solve for n:

$$\frac{(n+4)!}{(n+2)!} = 6$$

2. How many ways can the letters of the word *angle* be ordered such that exactly one letter is in its original position? For example, *nagel* is one such order, while *ganel*, *alnge*, and *gnale* are not.

3. Find the number of positive integers less than 1000 in which the sum of the digits of the number is 8. (e.g. 71, 521, 440)

ANSWERS

(1 pt) 1. ______________________

(2 pts) 2. ______________________

(3 pts) 3. ______________________

Groton-Dunstable, AMSA, Mass Academy

Worcester County Mathematics League
Varsity Meet 3 - January 22, 2020
Round 5 - Analytic Geometry

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. Consider points $P(0,-3), Q(-2,2), A(2,5)$, and $B(4,k)$ where k is a real number. Find the value of k that makes $\overline{PQ} \perp \overline{AB}$.

2. What is the midpoint of the segment whose endpoints are the points of intersection of $y = 2x+1$ and $y = x^2+12x+22$?

3. Given two ellipses with equations
$$4x^2 + 9(y-k)^2 = 36$$
and
$$9x^2 + y^2 + 10y = -16$$
find all values of k so the ellipses intersect at exactly one point.

ANSWERS

(1 pt) 1. ______________________

(2 pts) 2. ______________________

(3 pts) 3. ______________________

Notre Dame Academy, Algonquin, Shrewsbury

Worcester County Mathematics League
Varsity Meet 3 - January 22, 2020
Team Round

All answers must be in simplest exact form in the answer section.
NO CALCULATORS ALLOWED

1. In ΔABC, $\measuredangle B = 90°$, $BC = 16$, and $AC = 20$. Consider ΔDEF such that $\Delta ABC \sim \Delta DEF$ with $\frac{DE}{AB} = 3$. Determine the value of $\sin F \cdot \cos F \cdot \tan F$.
2. The function $f(x) = \frac{5x-2}{4x-a}$ is the same as its inverse for some number a. Find a.
3. Given $f(x-1) = x^2 + 3x + 5$, find where $f(x+1)$ crosses the y-axis.
4. Find n if
$$(3!)(5!)(7!) = n!$$
5. A circle is tangent to the x-axis, y-axis, and the line $y = -2x + 2$. Determine the radius of the circle.
6. Find all values of $\theta \in [0, 2\pi)$ such that
$$2\sin\theta\tan\theta - \tan\theta - 1 + 2\sin x = 0$$
7. If x is the average of m and 9, y is the average of $2m$ and 15, and z is the average of $3m$ and 18, what is the average of x, y, and z in terms of m?
8. There are 16 unique complex solutions to the equation $z^{16} + 1 = 0$. What is the maximum possible distance between any two of these solutions?
9. For some base B,
$$(15_B)^2 = 251_B$$
Find B.

Worcester County Mathematics League
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Team Round Answer Sheet

ANSWERS

1. ______________________________

2. ______________________________

3. ______________________________

4. ______________________________

5. ______________________________

6. ______________________________

7. ______________________________

8. ______________________________

9. ______________________________

Notre Dame Academy, Worcester Academy, Shrewsbury, Wachusett, Shrewsbury, Quaboag, Notre Dame Academy, Bancroft, Worcester Academy

Worcester County Mathematics League
Varsity Meet 3 - January 22, 2020
Answer Key

Round 1 - Similarity and Pythagorean Theorem

1. $\frac{20}{3}$ or $6\frac{2}{3}$ or $6.6\overline{6}$
2. $\sqrt{65}$
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Round 2 - Algebra I

1. 9
2. $(4,6)$ and $(-4,10)$
3. 5 and 1

Round 3 - Functions

1. 64
2. $-\frac{7}{3}$ (note: negative) or $-2\frac{1}{3}$ or $-2.3\overline{3}$
3. 12

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1. $\frac{9}{25}$ or 0.36
2. 5
3. 15
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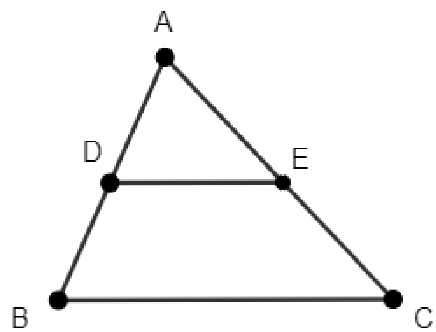
5. $\frac{3-\sqrt{5}}{2}$
6. $\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}$
7. $m+7$
8. 2
9. 8

Round 1 - Similarity and Pythagorean Theorem

1. Given ΔABC with $\overline{DE} \parallel \overline{BC}$, $AB = 8$, $AD = 6$, and $DE = 5$, find BC.

Solution:

Since $\overline{DE} \parallel \overline{BC}$, this means that two triangles are similar: $\Delta ADE \sim \Delta ABC$. From this we can set up a proportion with three of the four known quantities as well as an unknown quantity.

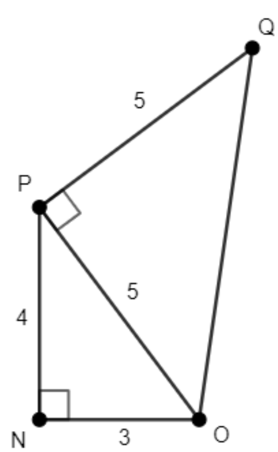


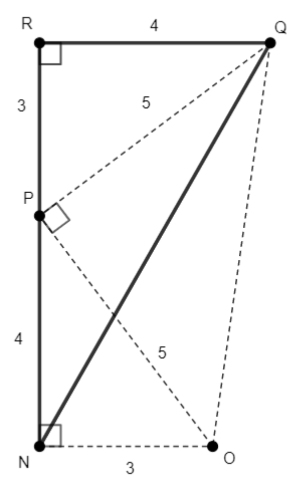
$$\frac{AB}{AD} = \frac{BC}{DE}$$
$$\frac{8}{6} = \frac{BC}{5}$$
$$\frac{40}{6} = BC$$
$$\boxed{\frac{20}{3} = BC}$$

2. Given the two right triangles in the picture shown to the right, how far apart are points N and Q?

Solution:

Fit ΔPRQ on top of the shape as shown below. Congruent to ΔONP, it has lengths of 3, 4, and 5, which is a common Pythagorean Triple and indicate that the triangle is a right triangle.

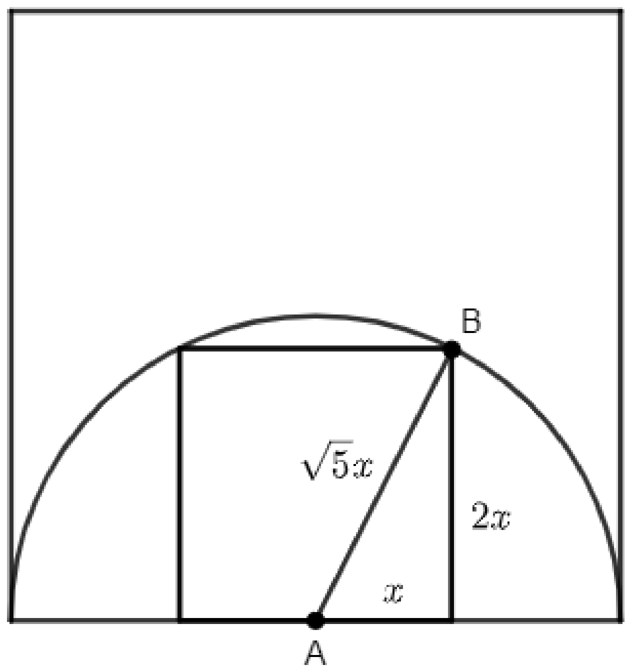




Right triangle ΔNRQ has hypotenuse NQ which has length $\sqrt{7^2 + 4^2} = \boxed{\sqrt{65}}$.

3. A square is inscribed in a semicircle which is inscribed in a larger square. Find the ratio of the smaller square's area to the larger square's area.

Solution: Draw a radius $\overline{AB}$ from the midpoint of the base of the small square to the upper right vertex. Let the distance from A to the corner be x, the square's side be equal to $2x$, and $AB = \sqrt{5}x$. $\overline{AB}$ is the radius of the semicircle and therefore half the large square's side length.



The small square has area $(2x)(2x) = 4x^2$ and the large square has area $(2\sqrt{5}x)(2\sqrt{5}x) = 20x^2$. The ratio of the areas, small to large, is $\dfrac{4x^2}{20x^2} = \boxed{\dfrac{1}{5}}$

Round 2 - Algebra I

1. Solve for x: $\sqrt{4x-11} = 2\sqrt{x} - 1$

Solution: Since the left side of the equation is a single radical, square both sides. Simplify and then isolate the other radical before squaring again.

$$\begin{aligned} \sqrt{4x-11} &= 2\sqrt{x} - 1 \\ 4x - 11 &= 4x - 4\sqrt{x} + 1 \\ -12 &= -4\sqrt{x} \\ 3 &= \sqrt{x} \\ \boxed{9} &= x \end{aligned}$$

2. Find all ordered pairs that satisfy the system below.

$$\begin{cases} \dfrac{2}{x} + \dfrac{1}{y} = \dfrac{x}{y} \\ \\ 2y + x = 16 \end{cases}$$

Solution: With the first equation, multiply through by xy:

$$2y + x = x^2$$

Replace $2y$ with $16 - x$ (from the second equation):

$$(16 - x) + x = x^2$$

$$16 = x^2$$

Knowing that $x = 4$ or $x = -4$, we plug both into the second equation to find the y values:

$$2y + 4 = 16 \Rightarrow y = 6$$

$$2y + (-4) = 16 \Rightarrow y = 10$$

The solutions are $\boxed{(4, 6) \text{ and } (-4, 10)}$.

3. Solve for x: $|x^2 + 3x - 10| = 6x$.

Solution: We begin by splitting into two separate equations.

$$\begin{aligned} x^2 + 3x - 10 &= 6x & \qquad x^2 + 3x - 10 &= -6x \\ x^2 - 3x - 10 &= 0 & \qquad x^2 + 9x - 10 &= 0 \\ (x-5)(x+2) &= 0 & \qquad (x+10)(x-1) &= 0 \end{aligned}$$

Our initial solutions are $x = 5, -2, -10, 1$ but we check and find that the negative values of x cause the absolute value to be equal to a negative number ($6(-2) = 12$ or $6(-10) = -60$) so our solutions are $\boxed{x = 5, 1}$.

Round 3 - Functions

1. If $f(x) = 3x + 5$ and $g(y) = 4 - y$, determine the value of

$$(f(3) - g(-2))^2$$

Solution:

$$(f(3) - g(-2))^2$$

$$((3(3) + 5) - (4 - (-2)))^2$$

$$(14 - 6)^2$$

$$\boxed{64}$$

2. Consider linear functions $f(x)$ and $g(x)$.

- $f(x)$ has a slope of $\frac{2}{3}$
- $g^{-1}(x) = 3f(x) - 1$
- $g(6) = 8$

Find $f(1)$.

Solution: Let's state what we know.

$$f(x) = \frac{2}{3}x + b$$

$$g^{-1}(x) = 3f(x) - 1 = 2x + 3b - 1$$

We also know, due to inverse functions, that

$$g^{-1}(8) = 6$$

and also that

$$g^{-1}(8) = 2(8) + 3b - 1 = 15 + 3b$$

Via the transitive property,

$$15 + 3b = 6$$

$$3b = -9$$

$$b = -3$$

and therefore

$$f(x) = \frac{2}{3}x - 3.$$

Now we can evaluate $f(1) = \frac{2}{3}(1) - 3 = \boxed{-\frac{7}{3}}$.

3. Function f has the property that for any two positive real numbers x and y that

$$f(xy) = f(x) + f(y).$$

Function g takes only positive values and has the property that for any real numbers x and y that

$$g(x + y) = g(x) \cdot g(y).$$

If $f(g(1)) = 3$, compute $f(g(4))$.

Solution: Let's break down the expression $f(g(4))$ into smaller parts via the function properties:

$$f(g(4))$$

$$f(g(2+2))$$

$$f(g(2) \cdot g(2))$$

$$f(g(2)) + f(g(2))$$

It looks like we have a new rule: $f(g(2m)) = f(g(m)) + f(g(m))$, so we use this to take a quick shortcut.

$$f(g(2)) + f(g(2))$$

$$f(g(1)) + f(g(1)) + f(g(1)) + f(g(1))$$

Each of these expressions is equal to 3, and so the sum of them is $\boxed{12}$.

Round 4 - Combinatorics

1. Solve for n:

$$\frac{(n+4)!}{(n+2)!} = 6$$

Solution: Expanding the factorial expressions until $n!$ and cancelling common factors from the numerator and denominator:

$$\frac{(n+4)!}{(n+2)!} = 6$$

$$\frac{(n+4)(n+3)(n+2)(n+1)n!}{(n+2)(n+1)n!} = 6$$

$$(n+4)(n+3) = 6$$

$$n^2 + 7n + 12 = 6$$

$$n^2 + 7n + 6 = 0$$

$$(n+6)(n+1) = 0$$

$$n = -6, -1$$

At first glance, these negative values don't seem to make sense, but one of them does.

$$n = -6 \Longrightarrow \frac{(n+4)!}{(n+2)!} = \frac{(-2)!}{(-4)!} \text{ (No!)}$$

$$\boxed{n = -1} \Longrightarrow \frac{(n+4)!}{(n+2)!} = \frac{(3)!}{(1)!} = 3 \cdot 2 = 6 \text{ (Yes!)}$$

2. How many ways can the letters of the word *angle* be ordered such that exactly one letter is in its original position? For example, *nagel* is one such order, while *ganel*, *alnge*, and *gnale* are not.

Solution: Assume the first letter is a so none of the other letters can be in their original positions. Pick the second letter g, and there are three ways to write the word starting with *ag*: *agnel, agenl, aglen.* There were two other choices for the second letter (l or e) and each of those choices begets three ways to finish off the word (*aleng, alneg, alegn, aelgn, aengl, aelng*) so there are $3 \cdot 3 = 9$ ways to order the word starting with a. The same counting principle applies to the other four letters (n in the starting position, etc...) leaving us with $5 \cdot 9 = \boxed{45}$ orders in total.

3. Find the number of positive integers less than 1000 in which the sum of the digits of the number is 8. (e.g. 71, 521, 440)

Solution: Breaking the problem up, let's look at each group of 100.

- **1-99**: There are a total of 9 numbers whose digits sum to 8. (80,71, 62, 53, 44, 35, 26, 17, 08)
- **100-199**: There are a total of 8 numbers whose digits sum to 8. (170, 161, 152, 143, 134, 125, 116, 107).
- **200-299**: There are a total of 7 numbers whose digits sum to 8. (260, 251, 243, 234, 225, 216, 206)

Continuing this process, each set of 100s has one fewer qualifying number, which has us at a total of $9+8+7+6+5+4+3+2+1 = \boxed{45}$ numbers. (Note that the 900s have none such numbers.)

Round 5 - Analytic Geometry

1. Consider points $P(0,-3), Q(-2,2), A(2,5)$, and $B(4,k)$ where k is a real number. Find the value of k that makes $\overline{PQ} \perp \overline{AB}$.

Solution: To be perpendicular the slopes of $\overline{PQ}$ and $\overline{AB}$ must be opposite reciprocals. Since the slope of $\overline{PQ}$ is $\frac{-2-(-3)}{-2-0} = -\frac{5}{2}$, the slope of $\overline{AB}$ must be $\frac{2}{5}$. We find that the value of k that satisfies this slope is

$$\frac{k-5}{4-2} = \frac{2}{5}$$

$$5k - 25 = 4$$

$$\boxed{k = \frac{29}{5} = 5.8}$$

2. What is the midpoint of the segment whose endpoints are the points of intersection of $y = 2x+1$ and $y = x^2 + 12x + 22$?

Solution: First, we find the points of intersection of the line and parabola.

$$2x + 1 = x^2 + 12x + 22$$

$$0 = x^2 + 10x + 21$$

$$0 = (x+7)(x+3)$$

Our x-coordinates are $x = -7, -3$ and we find the y-coordinates by plugging those x-coordinates back into the linear equation.

$$x = -3 \Longrightarrow 2(-3) + 1 = -5$$

$$x = -7 \Longrightarrow 2(-7) + 1 = -13$$

Then we average the coordinates to find the midpoint.

$$\left(\frac{(-3)+(-7)}{2}, \frac{(-5)+(-13)}{2}\right) = \left(\frac{-10}{2}, \frac{-18}{2}\right) = \boxed{(-5,-9)}$$

3. Given two ellipses with equations

$$4x^2 + 9(y-k)^2 = 36$$

and

$$9x^2 + y^2 + 10y = -16$$

find all values of k so the ellipses intersect at exactly one point.

Solution: First, we can determine the location and shape of each ellipse. The first ellipse, in standard form, is

$$\frac{x^2}{9} - \frac{(y-k)^2}{4} = 1$$

and the second is

$$9x^2 + y^2 + 10y = -16$$

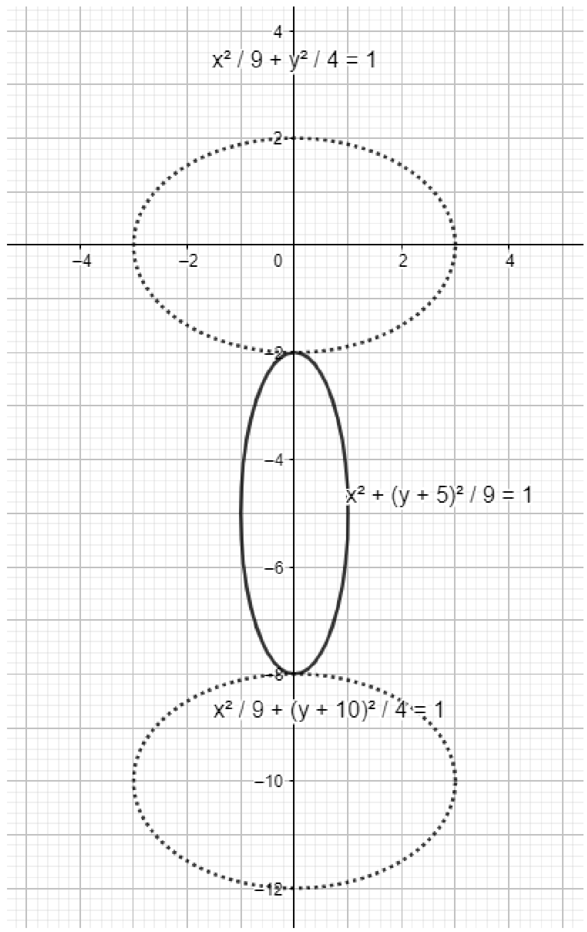
$$9x^2 + y^2 + 10y + 25 = 9$$

$$9x^2 + (y+5)^2 = 9$$

$$\frac{x^2}{1} + \frac{(y+5)^2}{9} = 1$$

The second ellipse is centered at $(0, -5)$ with a width of 2 and a height of 6.

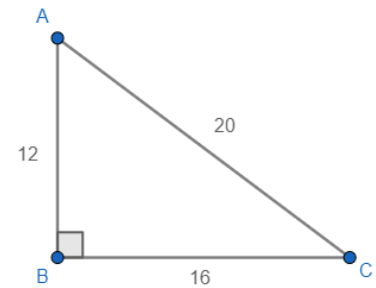
The first ellipse is centered at $(0, k)$ with a width of 6 and a height of 4. In order for these ellipses to intersect, the first ellipse can either be placed above or below the second ellipse by 2 units, meaning our values for k are either $\boxed{0 \text{ or } -10}$ (see diagram).



Team Round

1. In ΔABC, $\measuredangle B = 90°$, $BC = 16$, and $AC = 20$. Consider ΔDEF such that $\Delta ABC \sim \Delta DEF$ with $\dfrac{DE}{AB} = 3$. Determine the value of $\sin F \cdot \cos F \cdot \tan F$.

Solution:



Consider ΔABC, shown at right. Since ΔDEF is similar, trigonometric functions of the corresponding angles have identical values. Therefore,

$$\sin F \cos F \tan F = \sin C \cos C \tan C = \frac{12}{20} \cdot \frac{16}{20} \cdot \frac{12}{16} = \frac{144}{400} = \boxed{\frac{9}{25}}.$$

2. The function $f(x) = \dfrac{5x-2}{4x-a}$ is the same as its inverse for some number a. Find a.

Solution: Using the traditional "swap x and y and solve for the new y":

$$y = \frac{5x-2}{4x-a}$$

$$x = \frac{5y-2}{4y-a}$$

$$4xy - ax = 5y - 2$$

$$4xy - 5y = ax - 2$$

$$y(4x-5) = ax - 2$$

$$y = \frac{ax-2}{4x-5}$$

Therefore, $\boxed{a = 5}$. Alternatively, note that the horizontal asymptote of a rational function is the vertical asymptote of its inverse, and vice versa. If the function is its own inverse, the two asymptotes must correspond to the same number. The horizontal asymptote of $f(x)$ is $y = \frac{5}{4}$, and its vertical asymptote is $x = \frac{a}{4}$. So $a = 5$.

3. Given $f(x-1) = x^2 + 3x + 5$, find where $f(x+1)$ crosses the y-axis.

Solution: Since $f(x+1)$ is just the graph of $f(x-1)$ shifted left two units,

$$f(x+1) = (x+2)^2 + 3(x+2) + 5.$$

Plugging in $x = 0$ to find the y-intercept:

$$f(1) = 4 + 6 + 5 = \boxed{15}$$

4. Find n if

$$(3!)(5!)(7!) = n!$$

Solution:

$$(3!)(5!)(7!) = n!$$

$$(3 \cdot 2 \cdot 1)(\cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(\cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = n!$$

We need to arrange for $n!$ to have factors of 1 through n. Since 7! accounts for 1 through 7, let's look for 8, 9, etc... from 3!5! until we can't produce any more. With 3, 2, 5, 4, 3, and 2 we can produce and 8 (2 and 4), a 9 (3 and 3) and a 10 (5 and 2). Therefore, $(3!)(5!)(7!) = \boxed{10}!$.

5. A circle is tangent to the x-axis, y-axis, and the line $y = -2x + 2$. Determine the radius of the circle.

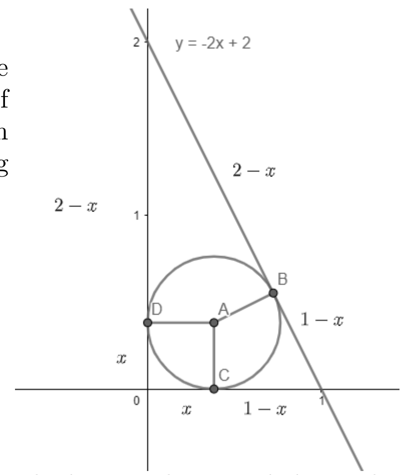
Solution:

Drawing radii from the center of the circle to each axis and the line, we find three pairs of congruent segments (Two-Tangent Theorem). Utilizing that the legs of the right triangle formed by the line are 2 units and 1 unit each, we can label each segment as x, $2 - x$, or $1 - x$, where x is the radius of the circle. Finally, noting that the hypotenuse of the triangle is $\sqrt{5}$,

$$(2 - x) + (1 - x) = \sqrt{5}$$

$$3 - 2x = \sqrt{5}$$

$$\boxed{x = \frac{3 - \sqrt{5}}{2}}$$



[Furthermore, using this approach, it's easily shown that the radius of a circle inscribed in a right triangle has radius $r = \frac{a+b-c}{2}$ where a and b are legs of the triangle and c is the hypotenuse. Cool.]

6. Find all values of $\theta \in [0, 2\pi)$ such that

$$2\sin\theta\tan\theta - \tan\theta - 1 + 2\sin x = 0$$

Solution:

$$2\sin\theta\tan\theta - \tan\theta - 1 + 2\sin\theta = 0$$

$$2\sin\theta\tan\theta + 2\sin\theta - \tan\theta - 1 = 0$$

$$2\sin\theta(\tan\theta + 1) - (\tan\theta + 1) = 0$$

$$(2\sin\theta - 1)(\tan\theta + 1) = 0$$

$$\sin\theta = \frac{1}{2} \qquad \tan\theta = -1$$

$$\theta = \frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{4}$$

7. If x is the average of m and 9, y is the average of $2m$ and 15, and z is the average of $3m$ and 18, what is the average of x, y, and z in terms of m?

Solution: We can make the following three equations

$$x = \frac{m+9}{2} \qquad y = \frac{2m+15}{2} \qquad z = \frac{3m+18}{2}$$
$$2x = m+9 \qquad 2y = 2m+15 \qquad 2z = 3m+18$$

and combine to achieve

$$2x + 2y + 2z = m + 9 + 2m + 15 + 3m + 18$$
$$2(x+y+z) = 6m + 42$$

Dividing both sides by 6 we find the average of x, y, and z to be

$$\frac{x+y+z}{3} = \boxed{m+7}$$

8. There are 16 unique complex solutions to the equation $z^{16} + 1 = 0$. What is the maximum possible distance between any two of these solutions?

Solution: Each of the solutions will be spaced evenly around the unit circle in the complex plane since each root will have magnitude 1. With an even number of solutions, there will be eight pairs of solutions directly opposite one another on the circle. This means the maximum distance between any pair of solutions will be equal to the diameter of the circle: $\boxed{2}$.

9. For some base B,

$$(15_B)^2 = 251_B$$

Find B.

Solution: The number 15_B can be written as $1B + 5$, so $(15_B)^2 = (B+5)^2 = B^2 + 10B + 25$. Similarly, 251_B can be written as $2B^2 + 5B + 1$. Equating these expressions:

$$2B^2 + 5B + 1 = B^2 + 10B + 25$$
$$B^2 - 5B - 24 = 0$$
$$(B-8)(B+3) = 0$$

Negative numbers aren't viable bases for numbers. Hence, $\boxed{B = 8}$.